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# A generic analysis for high-temperature power-law deformation: the case of linear ln(strain rate)–ln(stress) relationship

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Isostructural low-stress high-temperature deformation of different classes of materials is often represented by a power law that connects the strain rate to the flow stress through a stress exponent. The temperature dependence of the rate of deformation is assumed to be exponential. In Mukherjee *et al*'s popular approach the temperature dependence of the stress exponent is ignored by assuming a mean value for the stress exponent for the temperature range of interest and the stress is normalized with respect to the elastic constant. In the approach adopted by the engineers the stress is normalized with respect to a reference stress and it is possible to take into account the temperature dependence of the stress exponent while evaluating the activation energy for the rate-controlling process. Experimental data pertaining to 27 systems drawn from metals and alloys, superalloys, ceramics, glass ceramics, metal-matrix composites and an intermetallic, have been analysed using the latter approach to determine an activation energy for the rate-controlling process. It is demonstrated that this is an accurate description of high-temperature power-law deformation and that it involves less numbers of empirical constants than the former approach.

## 1. Rate equations

Low-stress, high-temperature deformation in different classes of materials is often described by a power law based on an assumption that the relevant mechanisms can be reduced to this form at least to a first-order approximation.

The materials scientists have favoured an equation of the type [1–4]

$$\dot{\varepsilon} = \left(\frac{AD_0Eb}{kT}\right)\left(\frac{b}{L}\right)^p\left(\frac{\sigma}{E}\right)^n \exp\left\{-\frac{Q}{kT}\right\} \qquad (1)$$

where $\dot{\varepsilon}$ is the tensile strain rate, $D_0$ is a frequency factor, $E$ is Young's modulus, $\boldsymbol{b}$ the Burgers vector, $k$ the Boltzmann constant, $T$ the absolute temperature of deformation, $L$ the grain size, $\sigma$ the tensile stress, $Q$ the activation energy for the rate-controlling deformation process, and $p, A$ and $n$ ($=1/m$, with $m$ the strain-rate sensitivity index) are material constants. For a single-phase material deforming by a unique rate-controlling process, this analysis requires eight empirical constants ($A, D_0, \boldsymbol{b}, p, n, Q$ and $a$ and $b$ in $E = a + bT$). The number rises to 10 for a two-phase material as in that case $E$ is computed from those of the constituent elements/phases using the rule of mixtures [5]. (Equation 1 may also be expressed in the shear mode.) As it is 'largely empirical', [1], Equation 1 is sometimes represented with small variations, see, for example, [6].

Although suggestions exist (a) that the physics behind Equation 1 is unclear [2–4] and (b) that Equation 1 will apply for a single mechanism only when $n$ is an integer [7], in practice the equation has been used without restrictions under conditions of diffusion and dislocation creep as well as structural superplasticity (see, for example, [1, 8]). The engineers prefer an equation of the form [3]

$$\dot{\varepsilon} = C\left(\frac{\nu}{kT}\right)\left(\frac{\sigma}{\sigma_c}\right)^n \exp\left(-\frac{Q}{kT}\right) \qquad (2)$$

where $C$ is a constant, $\nu$ the thermal vibration frequency is equal to either $(kT/h)$, with $h$ the Planck constant [9] or $10^{13}\ \mathrm{s}^{-1}$ [10, 11] and $\sigma_c$ is a reference stress. Both Equations 1 and 2 assume that the stress and temperature dependences of the flow rate can be expressed in the variable separable form. The main difference is that in Equation 1 the stress is normalized with respect to $E$ but in Equation 2 with respect to $\sigma_c$. A detailed analysis of the experimental data pertaining to different classes of materials [2–4] has revealed that the "constant of proportionality" in Equation 2 is approximately a constant, but it varies over a wide range in Equation 1.

TABLE I Different systems analysed, together with their composition and the temperature ranges over which the stress–strain rate data were taken

| Common Name | Composition (wt %) | Type[a] | Temperature range |  | Reference |
|---|---|---|---|---|---|
|  |  |  | Max. (K) | Min. (K) |  |
| 9608-Glass | 69.9$SiO_2$, 17.9$Al_2O_3$, 2.7$LiO_2$, 4.7$TiO_2$ | C | 1423 | 1373 | [20] |
| 0329-Glass | 64.6$SiO_2$, 19.9$Al_2O_3$, 3.6$LiO_2$, 4.3$TiO_2$ | C | 1386 | 1323 | [21] |
| 3YTZP | $ZrO_2$ +3$Y_2O_3$ | C | 1723 | 1523 | [21] |
| 4YTZP | $ZrO_2$ +4$Y_2O_3$ | C | 1723 | 1523 | [21] |
| 52YTZP | $ZrO_2$ +5.2$Y_2O_3$ | C | 1923 | 1623 | [22] |
| 6YTZP | $ZrO_2$ +6$Y_2O_3$ | C | 1723 | 1523 | [21] |
| 12CETZP | $ZrO_2$ +12$CeO_2$ | C | 1673 | 1523 | [21] |
| TZP20A | $ZrO_2$ +20(vol)%$Al_2O_3$ | C | 1723 | 1523 | [23] |
| 3YTZP40A | $ZrO_2$ +3$Y_2O_3$ +40(vol)%$Al_2O_3$ | C | 1723 | 1523 | [24] |
| 3YTZP60A | $ZrO_2$ +3$Y_2O_3$ +60(vol)%$Al_2O_3$ | C | 1723 | 1523 | [24] |
| 3YTZP80A | $ZrO_2$ +3$Y_2O_3$ +80(vol)%$Al_2O_3$ | C | 1723 | 1523 | [24] |
| 42YTZP20A | $ZrO_2$ +4.2$Y_2O_3$ +20(vol)%$Al_2O_3$ | C | 1923 | 1723 | [25] |
| $BaTiO_3$ | Ba Ti 3%O | C | 1523 | 1423 | [26] |
| SPINEL | Al–4Cu–1.5Mg–1.1C–0.8O | C | 1886 | 1723 | [27] |
| MA754 | Ni–20Cr–0.5Ti–0.3Al–0.6$Y_2O_3$ | SA | 1373 | 1173 | [28] |
| IN100 | Ni–12.38Cr–3.5Mo–18.5Co–4.3Ti–5Al | SA | 1373 | 1323 | [29] |
| Cu | 99.9% pure | R | 973 | 623 | [30] |
| Mg | 99.9% pure | R | 820 | 473 | [31] |
| W | 99.9% pure | R | 3073 | 1873 | [32] |
| Comp-s10 | Al-10$Al_2O_3$ | MMC | 823 | 296 | [33] |
| Comp-s14 | Al-14$Al_2O_3$ | MMC | 823 | 296 | [33] |
| Comp-s7 | Al-7$Al_2O_3$ | MMC | 773 | 296 | [33] |
| Comp-s4 | Al-4$Al_2O_3$ | MMC | 823 | 296 | [33] |
| WC-Co | WC–13Co | MMC | 1473 | 1373 | [34] |
| $Ni_3Si$ | Ni–9Si–3.1V–2Mo | I | 1373 | 1323 | [35] |
| Al–Cu | Al–33Cu eutectic | SP | 813 | 713 | [36] |
| Pb-Sn | Sn–38Pb eutectic | SP | 453 | 323 | [37] |

[a] C, ceramic; R, creep in a metal; SA, superalloy; MMC, metal matrix composite; I, intermetallic; SP, superplastic metallic alloy.

TABLE II Computed values of the true energy of activation for the different systems analysed. The activation energy values quoted by previous workers and the methods used by them for calculating these values are also listed

| System | Reported activation energy (kJ $mol^{-1}$) | Type of evaluation | Presently computed activation energy (kJ $mol^{-1}$) |  |
|---|---|---|---|---|
|  |  |  | $\nu = kT/h$ | $\nu = 10^{13}$ $s^{-1}$ |
| 9608-Glass | 635 ±35 | $Q_\sigma$[a] | 631 | 643 |
| 0329-Glass | 707 | $Q_\sigma$ | 800 | 811 |
| 3YTZP | 480 | $Q_\sigma$ | 468 | 481 |
| 4YTZP | 480 | $Q_\sigma$ | 412 | 425 |
| 52YTZP | 720 | Dorn[b] | 502 | 517 |
| 6YTZP | 491 | $Q_\sigma$ | 498 | 511 |
| 12CETZP | 498 | $Q_\sigma$ | 491 | 504 |
| TZP20A | 600 | Dorn | 577 | 590 |
| 3YTZP40A | 500 | $Q_\sigma$ | 680 | 693 |
| 3YTZP60A | 700 | $Q_\sigma$ | 737 | 750 |
| 3YTZP80A | 700 | $Q_\sigma$ | 749 | 762 |
| 42YTZP20A | NR[c] | – | 713 | 728 |
| $BaTiO_3$ | 800–1200 | $Q_\sigma$ | 1072 | 1084 |
| Spinel | 460 ±50 | $Q_\sigma$ | 429 | 444 |
| MA754 | NR | – | 611 | 622 |
| IN100 | 330 | $Q_\sigma$ | 644 | 655 |
| Cu | NR | – | 167 | 174 |
| Mg | 135 ±10 | Dorn | 163 | 168 |
| W | NR | Dorn | 611 | 632 |
| Comp-s10 | ⩽ 250 | Dorn | 196 | 201 |
| Comp-s14 | ⩽ 250 | Dorn | 284 | 289 |
| Comp-s7 | 45–150 | Dorn | 102 | 106 |
| Comp-s4 | ⩽ 250 | Dorn | 203 | 208 |
| WC–Co | 460 | $Q_\sigma$ | 454 | 466 |
| $Ni_3Si$ | 555 | $Q_\sigma$ | 556 | 567 |
| Al–Cu | 130 | Dorn | 132 | 138 |
|  | 253 | Dorn |  |  |
| Pb–Sn | 42 ±8 | Dorn | 32 | 35 |
|  | 92.5 ±13.5 | Dorn | 68 | 71 |

[a] $Q_\sigma$ = Activation energy at a constant stress = $(\partial \ln \dot{\varepsilon} / \partial 1/T)_\sigma$

[b] Dorn: Dorn analysis using a modulus compensated stress; Equation 1.

[c] NR, not reported.

TABLES III–XXIX Strain-rate estimates for the various systems analysed. The appropriate values of the activation energy for the rate-controlling process for the different systems are available in Table II. The predictions are based on Equation 3a: $\ln\dot{\varepsilon} = \ln C_1 + n\ln\sigma - n\ln\sigma_c - Q/RT$ (see text). Identical $\ln\dot{\varepsilon}$ values are predicted if Equation 3b, along with the corresponding values of activation energy, is used (see text)

TABLE III Common name 9608-Glass; class of material: ceramic.

$\mathrm{Ln}\,C_1 = 49.94$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln\sigma_c = 10.69$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -0.0987 + 1581.34/T$

| Temperature (K) | $\mathrm{Ln}\,\sigma$ (observed) ($\sigma$ in MPa) | $\mathrm{Ln}\,\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | $\mathrm{Ln}\,\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1373 | 7.611 | − 8.604 | − 8.589 | 0.17 |
| | 8.508 | − 7.666 | − 7.645 | 0.27 |
| 1386 | 7.195 | − 8.547 | − 8.471 | 0.88 |
| | 8.050 | − 7.638 | − 7.581 | 0.75 |
| 1398 | 6.362 | − 8.730 | − 8.827 | 1.11 |
| | 8.122 | − 6.943 | − 7.009 | 0.96 |
| 1410 | 6.108 | − 8.483 | − 8.583 | 1.18 |
| | 7.606 | − 6.947 | − 7.051 | 1.50 |
| 1423 | 5.614 | − 8.646 | − 8.544 | 1.17 |
| | 8.025 | − 6.201 | − 6.104 | 1.57 |

TABLE IV Common name 0329-Glass; class of material: ceramic.

$\mathrm{Ln}\,C_1 = 65.23$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln\sigma_c = 6.91$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -93.98 + 255\,516.7/T - 1.72\times10^8/T^2$.

Linear fit: $n_{\mathrm{pred}} = -0.34 + 1910.844/T$

| $T$ (K) | $\mathrm{Ln}\,\sigma$ (observed) ($\sigma$ in MPa) | $\mathrm{Ln}\,\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | $\mathrm{Ln}\,\dot{\varepsilon}$ (predicted) for best fit ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1323 | 5.492 | − 9.014 | − 9.045 | 0.34 | 0.77 |
| | 6.244 | − 8.207 | − 8.239 | 0.39 | 0.62 |
| 1336 | 4.957 | − 9.005 | − 8.950 | 0.62 | 0.77 |
| | 5.738 | − 8.152 | − 8.097 | 0.68 | 0.78 |
| 1348 | 4.396 | − 8.942 | − 8.925 | 0.19 | 0.80 |
| | 5.691 | − 7.535 | − 7.510 | 0.34 | 0.69 |
| 1360 | 4.291 | − 8.283 | − 8.377 | 1.14 | 0.47 |
| | 5.799 | − 6.668 | − 6.747 | 1.19 | 0.83 |
| 1373 | 4.255 | − 7.725 | − 7.767 | 0.75 | 0.92 |
| | 5.335 | − 6.564 | − 6.532 | 0.48 | 0.60 |
| 1386 | 4.276 | − 6.861 | − 6.869 | 0.12 | 1.16 |
| | 5.166 | − 5.975 | − 5.973 | 0.03 | 0.76 |

## 2. Activation energy

In Equation 1 the activation energy, $Q$, is determined for a material of constant grain size from the slope of an $\ln(\dot{\varepsilon}TE^{n-1})$ versus $1/T$ plot. Here $n$ is given a constant value in the stress and temperature ranges of interest. Often this is not justified by the experimental results. It would be difficult to justify this assumption as a general case also from theory [12, 13].

According to Sohal and Pearce [14] the method of estimating, for use in Equation 1, the value of $E$ of an alloy from the modulus values of the constituent phases/elements and the rule of mixtures [5], could lead to large errors in the value of $E$ employed. This conclusion is based on their experimental results for a Zn–Al eutectoid alloy and 60–40 brass. At a more fundamental level, in the case of high-temperature creep and superplasticity, where diffusion and annealing effects are not unimportant, the assumption that $\sigma \propto E$ (which is the basis for the normalization with respect to $E$ to obtain a "non-dimensional" stress) may not be valid for the following reasons.

(a) In diffusion creep mechanisms, e.g. Nabarro–Herring and Coble creep, the flow stress varies exponentially with temperature. The temperature compensation for $E$ in Equation 1, on the other hand, is often

TABLE V Common name 3YTZP; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 22.88$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 2.15$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -27.69 + 94\,276.14/T - 7.50 \times 10^7/T^2$.

Linear fit: $n_{\mathrm{pred}} = 0.77 + 2065.180/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) for best fit ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 2.385 | − 9.553 | − 9.373 | 1.88 | 2.02 |
| | 3.115 | − 8.229 | − 7.974 | 3.09 | 3.73 |
| | 3.807 | − 6.814 | − 6.650 | 2.42 | 3.74 |
| 1673 | 2.779 | − 9.327 | − 9.526 | 2.13 | 2.28 |
| | 3.066 | − 8.721 | − 8.944 | 2.55 | 2.78 |
| | 3.481 | − 7.963 | − 8.101 | 1.73 | 2.10 |
| 1623 | 2.762 | − 10.523 | − 10.555 | 0.29 | 0.60 |
| | 3.528 | − 8.867 | − 8.949 | 0.92 | 1.76 |
| | 4.006 | − 7.807 | − 7.946 | 1.79 | 3.07 |
| 1573 | 3.069 | − 10.932 | − 10.996 | 0.59 | 0.86 |
| | 3.782 | − 9.407 | − 9.489 | 0.87 | 1.43 |
| | 4.410 | − 8.184 | − 8.159 | 0.30 | 0.59 |
| 1523 | 3.103 | − 12.268 | − 12.142 | 1.03 | 1.45 |
| | 3.799 | − 10.823 | − 10.697 | 1.17 | 1.98 |
| | 4.199 | − 9.975 | − 9.871 | 1.05 | 2.15 |

TABLE VI Common name 4YTZP; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 28.33$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 7.52$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = 1.5212 + 434.32/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1723 | 0.956 | − 12.203 | − 12.054 | 1.22 |
| | 2.195 | − 9.992 | − 9.857 | 1.35 |
| | 3.127 | − 8.296 | − 8.204 | 1.10 |
| | 3.962 | − 6.882 | − 6.722 | 2.31 |
| 1673 | 1.362 | − 12.209 | − 12.239 | 0.24 |
| | 2.391 | − 10.301 | − 10.407 | 1.02 |
| | 3.528 | − 8.342 | − 8.382 | 0.48 |
| | 4.331 | − 6.909 | − 6.952 | 0.63 |
| 1623 | 1.322 | − 13.134 | − 13.272 | 1.06 |
| | 2.146 | − 11.630 | − 11.798 | 1.45 |
| | 3.067 | − 9.944 | − 10.151 | 2.08 |
| | 4.037 | − 8.278 | − 8.415 | 1.67 |
| 1573 | 1.713 | − 13.593 | − 13.591 | 0.01 |
| | 2.785 | − 11.624 | − 11.665 | 0.35 |
| | 3.798 | − 9.807 | − 9.844 | 0.39 |
| | 4.536 | − 8.555 | − 8.517 | 0.43 |
| 1523 | 1.345 | − 15.445 | − 15.343 | 0.66 |
| | 2.310 | − 13.728 | − 13.601 | 0.93 |
| | 3.478 | − 11.548 | − 11.489 | 0.50 |
| | 4.378 | − 9.973 | − 9.864 | 1.09 |

done through a linear fit [5]. Evidently, for these cases ($\sigma/E$) is *not* non-dimensional.

(b) In some dislocation models, e.g. climb-controlled steady-state creep of pure metals [15], the flow stress varies as $E^{1/N}$, with $N > 1$ ($1/N = 2/3$ for the above example). Regardless of the temperature dependences of $\sigma$ and $E$, for this case ($\sigma/E$) will *not* be non-dimensional.

(c) In some other models (see, for example, [15, 16]) the flow stress is made up of a short-range thermal component and a long-range (athermal) component. The constitutive equations in these cases are rather complex and again ($\sigma/E$) will *not* be non-dimensional.

For these cases, even within the stress–temperature space in which a single mechanism is rate controlling,

TABLE VII Common name 52YTZP; class of material: ceramic.

Ln $C_1 = 24.93$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 2.76$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = 162.33 - 546\,270/T + 4.7 \times 10^8/T^2$.

Linear fit: $n_{\mathrm{pred}} = 11.117\,35 - 12\,123.0/T$.

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1623 | 3.319 | − 10.537 | − 9.943 | 5.63 | 2.45 |
| | 3.555 | − 8.220 | − 8.975 | 9.18 | 4.14 |
| | 4.245 | − 6.349 | − 6.139 | 3.31 | 4.50 |
| 1723 | 2.604 | − 10.518 | − 10.638 | 1.14 | 0.15 |
| | 3.301 | − 8.221 | − 8.167 | 0.65 | 1.95 |
| | 3.775 | − 6.363 | − 6.490 | 1.98 | 6.04 |
| 1823 | 2.228 | − 10.525 | − 10.317 | 1.98 | 1.94 |
| | 2.705 | − 8.248 | − 8.386 | 1.67 | 4.20 |
| | 3.252 | − 6.351 | − 6.174 | 2.79 | 6.61 |
| 1923 | 2.199 | − 9.374 | − 9.422 | 0.52 | 3.17 |
| | 2.692 | − 7.064 | − 6.805 | 3.66 | 13.66 |
| | 2.717 | − 6.365 | − 6.673 | 4.84 | 7.52 |

TABLE VIII Common name 6YTZP; class of material: ceramic.

Ln $C_1 = 21.67$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 1.90$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = 16.98 - 51\,072.4/T + 4.18 \times 10^7/T^2$.

Linear fit: $n_{\mathrm{pred}} = 1.022\,119 + 617.8982/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 2.122 | − 12.765 | − 12.754 | 0.08 | 0.03 |
| | 3.040 | − 11.429 | − 11.460 | 0.27 | 0.58 |
| | 3.974 | − 10.132 | − 10.142 | 0.10 | 0.72 |
| 1673 | 2.343 | − 13.681 | − 13.495 | 1.36 | 1.40 |
| | 3.036 | − 12.751 | − 12.540 | 1.66 | 1.76 |
| | 3.970 | − 11.505 | − 11.252 | 2.20 | 2.42 |
| 1623 | 2.309 | − 14.453 | − 14.647 | 1.34 | 1.25 |
| | 3.474 | − 12.780 | − 13.048 | 2.10 | 1.73 |
| | 4.107 | − 11.882 | − 12.179 | 2.51 | 1.94 |
| 1573 | 2.080 | − 16.022 | − 16.130 | 0.67 | 0.65 |
| | 2.707 | − 15.114 | − 15.253 | 0.92 | 0.82 |
| | 3.400 | − 14.154 | − 14.286 | 0.93 | 0.74 |
| | 4.044 | − 13.356 | − 13.386 | 0.22 | 0.07 |
| 1523 | 2.076 | − 17.505 | − 17.373 | 0.75 | 0.72 |
| | 2.715 | − 16.607 | − 16.442 | 0.99 | 0.84 |
| | 3.412 | − 15.557 | − 15.425 | 0.84 | 0.55 |
| | 4.002 | − 14.690 | − 14.565 | 0.85 | 0.42 |

one may obtain different activation energies merely because $\sigma$ and $E$ vary differently with temperature or have no linear interdependence, i.e. a spurious result may be obtained because of faulty normalization.

Thus, it would be unwise to ignore Basinski's advice [17] that $\sigma$ should be normalized with respect to $E$ only when diffusion and annealing effects are unimportant. So far as we are able to see, the above statements are valid regardless of whether the steady-state strain rate–stress data are obtained in constant load, constant stress or constant strain-rate tests.

On the other hand, $(\sigma/\sigma_c)$ will always be non-dimensional, as the functional dependence of both $\sigma$ and $\sigma_c$ (on $T, L$ and $\dot{\varepsilon}$) will be the same. Further, in Equation 2, consistent with the experimental results [18], $n$ can be taken to be temperature dependent [4]. The narrow temperature interval within which $n$ is practically independent of temperature will be considered as a special case. Thus, in our view, for high-temperature

TABLE IX Common name 12CETZP; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 22.96$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 2.25$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -41.04 + 136\,025.3/T - 1.1 \times 10^8/T^2$.

Linear fit: $n_{\mathrm{pred}} = 1.855\,269 - 914.277/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1673 | 3.024 | − 11.355 | − 11.347 | 0.07 | 0.36 |
| | 3.948 | − 10.180 | − 10.178 | 0.03 | 0.74 |
| | 4.272 | − 9.789 | − 9.767 | 0.22 | 1.10 |
| 1623 | 2.791 | − 12.686 | − 12.694 | 0.06 | 0.23 |
| | 3.433 | − 11.823 | − 11.839 | 0.14 | 0.53 |
| | 4.112 | − 10.899 | − 10.935 | 0.32 | 0.99 |
| 1573 | 2.739 | − 13.917 | − 13.925 | 0.06 | 0.22 |
| | 3.456 | − 13.013 | − 12.980 | 0.26 | 0.16 |
| | 4.098 | − 12.150 | − 12.133 | 0.15 | 0.54 |
| 1523 | 2.970 | − 14.926 | − 14.929 | 0.02 | 0.18 |
| | 3.485 | − 14.304 | − 14.304 | 0.00 | 0.36 |
| | 4.122 | − 13.521 | − 13.530 | 0.07 | 0.50 |

TABLE X Common name TZP20A; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 28.52$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 1.88$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = 1251.76 - 6047\,152/T + 9.7 \times 10^9/T^2 - 5.2 \times 10^{12}/T^3$.

Linear fit: $n_{\mathrm{pred}} = 3.346\,875 - 2161.59/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 2.759 | − 9.779 | − 9.855 | 0.78 | 1.44 |
| | 3.626 | − 7.909 | − 7.976 | 0.85 | 2.49 |
| | 3.985 | − 7.166 | − 7.199 | 0.46 | 2.64 |
| | 4.473 | − 6.065 | − 6.141 | 1.25 | 4.43 |
| 1673 | 1.705 | − 13.311 | − 13.305 | 0.05 | 0.08 |
| | 4.181 | − 8.394 | − 8.452 | 0.69 | 1.91 |
| 1623 | 2.020 | − 14.056 | − 13.964 | 0.66 | 0.70 |
| | 4.166 | − 9.891 | − 9.722 | 1.70 | 2.60 |
| 1573 | 2.352 | − 14.726 | − 14.631 | 0.64 | 0.41 |
| | 4.227 | − 10.855 | − 10.801 | 0.50 | 1.05 |
| 1523 | 2.791 | − 15.189 | − 15.302 | 0.74 | 0.67 |
| | 4.139 | − 12.615 | − 12.721 | 0.84 | 0.62 |

creep and superplasticity, normalization with respect to $\sigma_c$ is preferable to normalization with reference to $E$.

## 3. A procedure for deriving an activation energy

In this paper, the case $n = f\ (L, T)$ (where $f$ is an appropriate function) is considered, i.e. for a material of constant grain size and temperature of deformation, $n$ has a unique value. The case $n = F(L, T, \sigma)$ (where $F$ is another appropriate function) is more complicated [19] and will be considered separately.

When $\nu = (kT/h)$ and $C_1 = (C/h)$, from Equation 2, it follows that

$$\ln \dot{\varepsilon} = \ln C_1 + n \ln \sigma - n \ln \sigma_c - \left(\frac{Q}{kT}\right) \quad (3a)$$

When $\nu = 10^{13}\ \mathrm{s}^{-1}$ and $C_2 = 10^{13}(C/k)$, Equation 2 yields

$$\ln \dot{\varepsilon} = \ln C_2 - \ln T + n \ln \sigma - n \ln \sigma_c - \left(\frac{Q}{kT}\right) \quad (3b)$$

Equation 3a for the isothermal case can be written as

TABLE XI Common name 3YTZP40A; class of material: ceramic.

$\text{Ln}\,C_1 = 39.45$ ($C_1$ in $\text{s}^{-1}$); $\ln\sigma_c = 3.73$ ($\sigma_c$ in MPa).

Best fit: $n_{\text{pred}} = 17.526 - 52\,731.0/T + 4.52\times10^7/T^2$.

Linear fit: $n_{\text{pred}} = 0.26 + 3184.317/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\text{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\text{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 1.026 | − 13.691 | − 13.799 | 0.79 | 0.14 |
| | 3.031 | − 9.553 | − 9.494 | 0.62 | 0.86 |
| | 4.041 | − 7.268 | − 7.326 | 0.80 | 0.94 |
| 1673 | 1.247 | − 14.982 | − 14.765 | 1.45 | 1.23 |
| | 3.579 | − 9.696 | − 9.737 | 0.43 | 0.45 |
| 1623 | 1.854 | − 14.743 | − 15.035 | 1.98 | 2.40 |
| | 3.958 | − 10.491 | − 10.415 | 0.71 | 0.79 |
| 1573 | 2.396 | − 15.837 | − 15.547 | 1.83 | 1.66 |
| | 4.197 | − 11.530 | − 11.458 | 0.62 | 0.70 |
| 1523 | 3.059 | − 15.710 | − 15.827 | 0.75 | 0.61 |
| | 4.268 | − 12.859 | − 12.939 | 0.62 | 0.76 |

TABLE XII Common name 3YTZP60A; class of material: ceramic.

$\text{Ln}\,C_1 = 43.95$ ($C_1$ in $\text{s}^{-1}$); $\ln\sigma_c = 4.41$ ($\sigma_c$ in MPa).

Best fit: $n_{\text{pred}} = 2098.394 - 1.0\times10^7/T + 1.7\times10^{10}/T^2 - 9.2\times10^{12}/T^3$.

Linear fit: $n_{\text{pred}} = 5.134\,265 - 4576.20/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\text{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\text{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 1.268 | − 15.040 | − 15.107 | 0.45 | 1.64 |
| | 4.098 | − 8.235 | − 8.254 | 0.24 | 0.45 |
| 1673 | 1.840 | − 15.197 | − 14.967 | 1.51 | 0.03 |
| | 3.942 | − 10.217 | − 10.116 | 0.99 | 0.57 |
| 1623 | 2.363 | − 15.387 | − 15.647 | 1.69 | 0.13 |
| | 4.088 | − 11.335 | − 11.451 | 1.02 | 0.69 |
| 1573 | 2.759 | − 16.576 | − 16.471 | 0.64 | 3.00 |
| | 4.170 | − 13.029 | − 12.995 | 0.26 | 0.69 |
| 1523 | 3.483 | − 16.036 | − 16.037 | 0.01 | 1.20 |
| | 4.267 | − 14.538 | − 14.530 | 0.06 | 0.15 |

$$\ln\dot{\varepsilon} = A_1 + B\ln\sigma \tag{4a}$$

where $A_1 = \ln C_1 - n\ln\sigma_c - (Q/kT)$ and $B = n$.

On the other hand, Equation 3b for the isothermal case can be represented as

$$\ln\dot{\varepsilon} = A_2 + B\ln\sigma \tag{4b}$$

with $A_2 = \ln C_2 - \ln T - n\ln\sigma_c - (Q/kT)$. $A_1$ and $A_2$ for different temperatures can be obtained from least squares fits between $\ln\dot{\varepsilon}$ and $\ln\sigma$ (isothermal plots). From Equations 4a and b one obtains, respectively, at different temperatures

$$A_{1i} = \ln C_1 - n_i\ln\sigma_c - \left(\frac{Q}{kT_i}\right) \tag{5a}$$

and

$$A_{2i} + \ln T_i = \ln C_2 - n_i\ln\sigma_c - \left(\frac{Q}{kT_i}\right) \tag{5b}$$

where the suffix $i$ indicates that the value corresponds to a given temperature $T_i$. In both Equations 5a and b there are three unknowns: $\ln\sigma_c, Q$ and $\ln C_1$ or $\ln C_2$, as the case may be. Using the method of least squares, it can be written that

$$\sum\left[A_{1i} - \ln C_1 + n_i\ln\sigma_c + \left(\frac{Q}{kT_i}\right)\right]^2$$

$$= \phi_1(\ln C_1, \ln\sigma_c, Q) = \text{minimum} \tag{6a}$$

TABLE XIII Common name 3YTZP80A; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 44.38$ ($C_1$ in $s^{-1}$); $\ln\sigma_c = 3.35$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = 79.47071 - 251038/T + 2.0 \times 10^8/T^2$.

Linear fit: $n_{\mathrm{pred}} = 2.083055 - 339.250/T$

| $T$ (K) | $\mathrm{Ln}\,\sigma$ (observed) ($\sigma$ in MPa) | $\mathrm{Ln}\,\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | $\mathrm{Ln}\,\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 1.738 | − 13.056 | − 13.223 | 1.35 | 1.60 |
| | 3.286 | − 10.028 | − 10.085 | 0.56 | 1.00 |
| | 4.103 | − 8.333 | − 8.422 | 1.06 | 0.63 |
| 1673 | 1.796 | − 14.399 | − 14.133 | 1.85 | 0.79 |
| | 4.131 | − 9.962 | − 9.881 | 0.81 | 0.68 |
| 1623 | 2.078 | − 15.103 | − 15.069 | 0.23 | 1.97 |
| | 4.085 | − 11.630 | − 11.601 | 0.25 | 0.08 |
| 1573 | 3.436 | − 14.423 | − 14.535 | 0.78 | 1.33 |
| | 4.253 | − 13.028 | − 13.080 | 0.40 | 0.46 |
| 1523 | 3.169 | − 17.203 | − 17.158 | 0.26 | 1.26 |
| | 4.450 | − 14.590 | − 14.589 | 0.01 | 0.09 |

TABLE XIV Common name 42YTZP20A; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 32.18$ ($C_1$ in $s^{-1}$); $\ln\sigma_c = -0.85$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -4.07902 + 11047.71/T$

| Temperature (K) | $\mathrm{Ln}\,\sigma$ (observed) ($\sigma$ in MPa) | $\mathrm{Ln}\,\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | $\mathrm{Ln}\,\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1723 | 2.722 | − 9.117 | − 9.234 | 1.29 |
| | 2.981 | − 8.533 | − 8.629 | 1.13 |
| | 3.280 | − 7.800 | − 7.933 | 1.70 |
| 1823 | 2.472 | − 8.459 | − 8.256 | 2.40 |
| | 2.826 | − 7.793 | − 7.556 | 3.05 |
| | 3.164 | − 7.111 | − 6.887 | 3.15 |
| | 3.292 | − 6.889 | − 6.632 | 3.72 |
| | 3.637 | − 6.231 | − 5.948 | 4.55 |
| 1923 | 2.279 | − 7.073 | − 7.180 | 1.51 |
| | 2.400 | − 6.884 | − 6.979 | 1.37 |
| | 2.769 | − 6.227 | − 6.364 | 2.20 |
| | 3.123 | − 5.652 | − 5.775 | 2.16 |

$$\sum\left[A_{2i} + \ln T_i - \ln C_2 + n_i \ln\sigma_c + \left(\frac{Q}{kT_i}\right)\right]^2$$

$$= \phi_2(\ln C_2, \ln\sigma_c, Q) = \text{minimum} \qquad \text{(6b)}$$

where $\phi_1$ and $\phi_2$ are appropriate functions of $\ln C_1, \ln\sigma_c$, $Q$ and $\ln C_2, \ln\sigma_c, Q$, respectively. Thus, following standard least squares procedure, the "best fit values" of $\ln C_1$ (or $\ln C_2$), $\ln\sigma_c$ and $Q$ can be obtained for each system.

## 4. Numerical procedure and results

Following a survey of the literature, the isostructural stress–strain rate data at different temperatures for 27 systems that represent high-temperature creep in metals and superalloys, superplasticity/anomalous ductility in ceramics, glass ceramics, metal matrix composites, metals (in which the reported $\ln\dot{\varepsilon}$–$\ln\sigma$ relationship was linear) and an intermetallic, were digitized and analysed. (Data concerning some more intermetallics were also generated and analysed under an Indo–American programme and this will be reported elsewhere.) Table I gives a list of the different systems analysed, their compositions, the temperature range over which the $\ln\dot{\varepsilon}$–$\ln\sigma$ data were digitized and the references from which the data have been obtained.

It is made clear at this stage (a) that the statement of the original authors that their results represent steady-state data has been accepted by us, and (b) that no effort was made to check if these results were obtained in constant load, constant stress or constant strain-rate tests, as the analysis presented

TABLE XV Common name $BaTiO_3$; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 77.67$ ($C_1$ in $s^{-1}$); $\ln \sigma_c = 2.44$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = -899.406 + 2631400/T - 1.9 \times 10^9/T^2$.

Linear fit: $n_{pred} = -13.7737 + 23903.94/T$

| $T$ (K) | Ln σ (observed) (σ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1423 | 2.318 | − 13.469 | − 13.241 | 1.70 | 1.24 |
| | 2.706 | − 12.272 | − 12.264 | 0.06 | 1.16 |
| | 3.022 | − 11.037 | − 11.468 | 3.91 | 1.23 |
| | 3.685 | − 9.798 | − 9.799 | 0.01 | 6.45 |
| | 4.075 | − 9.107 | − 8.819 | 3.16 | 12.29 |
| 1448 | 2.311 | − 11.872 | − 11.756 | 0.98 | 1.28 |
| | 2.699 | − 10.548 | − 10.586 | 0.37 | 1.06 |
| | 2.980 | − 9.770 | − 9.739 | 0.31 | 1.25 |
| 1473 | 2.332 | − 10.249 | − 10.177 | 0.70 | 1.24 |
| | 2.712 | − 8.899 | − 9.052 | 1.72 | 3.27 |
| | 2.980 | − 8.391 | − 8.256 | 1.60 | 1.68 |
| 1498 | 1.637 | − 10.151 | − 10.332 | 1.79 | 0.02 |
| | 2.315 | − 8.573 | − 8.697 | 1.45 | 1.12 |
| 1523 | 1.615 | − 8.256 | − 8.147 | 1.32 | 3.79 |
| | 2.274 | − 7.293 | − 7.218 | 1.02 | 0.14 |

TABLE XVI Common name spinel; class of material: ceramic.

$\mathrm{Ln}\, C_1 = 20.78$ ($C_1$ in $s^{-1}$); $\ln \sigma_c = 4.81$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = 6990.269 - 3.8 \times 10^7/T + 6.9 \times 10^{10}/T^2 - 4.2 \times 10^{13}/T^3$.

Linear fit: $n_{pred} = -5.01802 + 12925.89/T$

| $T$ (K) | Ln σ (observed) (σ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 1723 | 4.224 | − 10.398 | − 10.481 | 0.80 | 1.82 |
| | 5.410 | − 7.670 | − 7.751 | 1.06 | 0.37 |
| 1716 | 3.991 | − 10.311 | − 10.344 | 0.32 | 2.13 |
| | 4.902 | − 7.963 | − 8.003 | 0.50 | 0.86 |
| 1825 | 3.580 | − 10.286 | − 10.006 | 2.72 | 2.78 |
| | 5.071 | − 7.228 | − 6.920 | 4.26 | 4.24 |
| 1856 | 3.412 | − 9.563 | − 9.533 | 0.31 | 1.54 |
| | 5.105 | − 6.449 | − 6.451 | 0.03 | 0.55 |
| 1886 | 3.062 | − 9.551 | − 9.747 | 2.05 | 2.14 |
| | 4.817 | − 6.349 | − 6.534 | 2.92 | 2.92 |

here is applicable to all the three types of testing. (It is interesting that Cahn and Haasen [15] have also concluded that the information obtained with regard to steady-state flow is rather similar whether it is generated in a constant stress or in a constant strain-rate test.)

The $\ln \dot{\varepsilon}$–$\ln \sigma$ data were regressed to obtain

$$\ln \dot{\varepsilon} = A_1 + B \ln \sigma \tag{7a}$$

or

$$\ln \dot{\varepsilon} = A_2 + B \ln \sigma \tag{7b}$$

depending on whether ν is taken as equal to $(kT/h)$ or $10^{13}\, s^{-1}$. As seen earlier, $B$ is equal to $n$ and its magnitude depends on the temperature and the system. For each system,

$$A_{1i} = \ln C_1 - n_i \ln \sigma_c - \left(\frac{Q}{kT_i}\right) \tag{8a}$$

$$A_{2i} + \ln T_i = \ln C_2 - n_i \ln \sigma_c - \left(\frac{Q}{kT_i}\right) \tag{8b}$$

$\ln C_1$ (or $\ln C_2$), $\ln \sigma_c$ and $Q$ are evaluated from Equation 8a and b as unique constants for each system in

TABLE XVII Common name MA754; class of material: superalloy.

Ln $C_1$ = 55.03 ($C_1$ in $s^{-1}$); ln $\sigma_c$ = 4.23 ($\sigma_c$ in MPa).

Best fit: $n_{\text{pred}} = 15.784\,13 - 9949.64/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1173 | 4.145 | −8.304 | −8.234 | 0.83 |
| | 4.345 | −6.712 | −6.777 | 0.98 |
| | 4.544 | −5.286 | −5.329 | 0.67 |
| 1273 | 3.476 | −8.328 | −8.703 | 4.50 |
| | 3.752 | −6.127 | −6.499 | 6.07 |
| | 3.871 | −5.451 | −5.553 | 1.88 |
| 1373 | 3.122 | −8.374 | −7.949 | 5.08 |
| | 3.259 | −7.011 | −6.780 | 3.29 |
| | 3.399 | −5.830 | −5.583 | 4.22 |
| | 3.472 | −5.273 | −4.956 | 6.01 |

TABLE XVIII Common name IN100; class of material: superalloy.

Ln $C_1$ = 3.81 ($C_1$ in $s^{-1}$); ln $\sigma_c$ = 3.81 ($\sigma_c$ in MPa).

Best fit: $n_{\text{pred}} = -5.769\,28 + 10\,988.3/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1373 | 2.760 | −9.379 | −9.439 | 0.64 |
| | 3.422 | −7.764 | −7.961 | 2.55 |
| | 3.798 | −7.080 | −7.120 | 0.57 |
| | 4.388 | −5.971 | −5.823 | 2.46 |
| 1348 | 3.393 | −9.381 | −9.133 | 2.65 |
| | 3.852 | −7.802 | −8.041 | 3.07 |
| | 4.193 | −7.096 | −7.229 | 1.88 |
| | 4.648 | −5.999 | −6.144 | 2.42 |
| 1323 | 3.988 | −8.693 | −8.773 | 0.94 |
| | 4.465 | −7.778 | −7.565 | 2.73 |
| | 4.718 | −7.043 | −6.922 | 1.71 |
| | 5.167 | −6.019 | −5.785 | 3.87 |
| | 5.310 | −5.504 | −5.421 | 1.51 |

a given microstructural condition and these are independent of both stress and temperature, $Q$ will also be independent of grain size. ln $C_1$ or ln $C_2$ (as the case may be) will depend on grain size. To keep the complexity in Equation 2 minimum, it is desirable to choose $\sigma_c$ to be a material constant, i.e. $\sigma_c$ should be independent of not only stress and temperature but also grain size. (Then the grain-size dependence in Equation 2 will be introduced through $C$.)

Two special cases, namely (a) $n$ is independent of temperature, and (b) $n$ is a linear function of $(1/T)$, are worthy of separate consideration. When $n$ is a constant, i.e. it is independent of $T$, it will not be possible to evaluate $C_1$ (or $C_2$) and $\sigma_c$ independently, and when $n$ is a linear function of $(1/T)$, $\sigma_c$ and $Q$ cannot be estimated independently if an attempt is made to calculate all these quantities *directly* from either Equation 8a or b.

It should be realized that experimentally the ln $\dot{\varepsilon}$–ln $\sigma$ relationship at constant grain size and temperature is obtained first from which the *numerical* values of $n$ can be determined. These values corresponding to different temperatures are then substituted in Equation 8a or b (when $n$ is a constant as well as when it is a linear function of $(1/T)$) to obtain $C_1$ (or $C_2$), $\sigma_c$ and $Q$ uniquely.

Using the procedure outlined earlier, the activation energy for all the systems were evaluated for both the cases: $\nu = (kT/h)$ and $\nu = 10^{13}\,s^{-1}$. The calculated values of the activation energy are reported in Table II. For comparison, the values of the activation energy reported by the original authors in the case of many systems as well as the procedures used by them, are also mentioned in Table II.

In the present procedure, from Equations 4a and b the values of the stress exponent $n$ $(=B)$ at the temperatures for which the experimental data have been chosen as input can be obtained. Using these values of $n$ and the corresponding temperatures of deformation on an absolute scale, "best fit" equations

TABLE XIX Common name, copper; class of material: (creep in) metal.

$\mathrm{Ln}\,C_1 = 12.72$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 3.41$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = 23.75752 - 33054.1/T + 1.38 \times 10^7/T^2$.

Linear fit: $n_{\mathrm{pred}} = 0.485751 + 3202.745/T$

| $T$ (K) | $\mathrm{Ln}\,\sigma$ (observed) ($\sigma$ in MPa) | $\mathrm{Ln}\,\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | $\mathrm{Ln}\,\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 973 | 1.782 | – 14.700 | – 15.058 | 2.44 | 4.31 |
| | 2.365 | – 12.192 | – 12.501 | 2.53 | 2.70 |
| | 3.198 | – 8.737 | – 8.846 | 1.25 | 0.24 |
| 923 | 1.826 | – 15.671 | – 15.647 | 0.19 | 2.37 |
| | 2.585 | – 12.492 | – 12.474 | 0.13 | 1.56 |
| | 2.533 | – 8.731 | – 8.523 | 2.38 | 2.08 |
| 873 | 2.017 | – 16.297 | – 15.896 | 2.46 | 1.41 |
| | 3.019 | – 11.918 | – 11.859 | 0.49 | 0.09 |
| | 3.990 | – 7.624 | – 7.942 | 4.18 | 3.24 |
| 823 | 2.269 | – 16.533 | – 16.246 | 1.74 | 0.86 |
| | 3.259 | – 12.317 | – 12.283 | 0.27 | 0.19 |
| | 4.303 | – 7.954 | – 8.104 | 1.89 | 2.32 |
| 773 | 2.477 | – 17.110 | – 17.107 | 0.02 | 2.71 |
| | 3.402 | – 13.319 | – 13.288 | 0.23 | 0.19 |
| | 4.233 | – 9.944 | – 9.855 | 0.89 | 5.01 |
| 723 | 2.935 | – 16.935 | – 17.184 | 1.47 | 2.69 |
| | 3.661 | – 14.026 | – 13.928 | 0.69 | 1.46 |
| | 4.121 | – 11.917 | – 11.866 | 0.43 | 3.01 |
| 673 | 3.300 | – 17.683 | – 17.684 | 0.00 | 0.05 |
| | 3.883 | – 14.716 | – 14.673 | 0.29 | 0.56 |
| | 4.389 | – 12.198 | – 12.064 | 1.09 | 1.76 |
| 623 | 3.817 | – 16.993 | – 16.942 | 0.30 | 1.34 |
| | 4.203 | – 14.229 | – 14.504 | 1.93 | 5.76 |
| | 4.657 | – 11.546 | – 11.638 | 0.80 | 8.22 |

between $n$ and $(1/T)$ can be obtained. (As $T \gg 1$, a polynomial fit in terms of $(1/T)$ is preferable to the one in terms of $T$, as in the former case there will be no problems of convergence and the truncation errors will also be considerably less than in the latter case.)

From the computed values of $\ln C_1$ (or $\ln C_2$), $\sigma_c$, the activation energy for a particular system, and the prediction equation for $n$ in terms of $(1/T)$ described above, the strain rates were calculated at different temperatures and for various stress levels. The percentage errors in the computed values of these strain rates were also determined. Tables III–XXIX list the values of $\ln C_1$, $\sigma_c$, $n$ as a function of $(1/T)$ and the errors in the calculated values of the strain rate for all the temperatures at which the $\ln \dot{\varepsilon}$–$\ln \sigma$ data were analysed for the systems chosen for this investigation. (Needless to say, for any other temperature within the chosen interval, the value of $n$ can be evaluated from the prediction equation in terms of $(1/T)$ and the strain rates estimated as before.) It is clear from Tables III–XXIX that the errors in the estimated values of the strain rate are very low, i.e. Equation 2 is a good description of the data concerning the 27 systems analysed in this paper. The "best fit" equation for the $n$ versus $(1/T)$ relation was linear for nine systems, quadratic for 14 systems and cubic for four systems (making a total of 27 systems). Even when a linear fit for the $n$ versus $(1/T)$ relationship was generated and used for the latter 18 systems, the strain rates estimated agreed very well with the experimental results in all but three systems in which the temperature range covered was of the order of 300–400 °C. Evidently, for these systems, $n$ versus $(1/T)$ fits of an order higher than linear, as indicated in Tables III–XXIX, will have to be employed.

It is conceivable that when the temperature interval was rather wide, there was a change in the rate-controlling mechanism and this is reflected as a non-linear dependence of $n$ on $(1/T)$. In the absence of detailed metallographic observations in this regard in the original papers, we are unable to consider this possibility further.

A generic analysis of the kind presented in this paper is based on an "archetypal rate equation" that, regardless of the details of the physical processes of deformation involved, is of considerable use in design.

## 5. Possible extension

If a grain-size dependence is introduced in Equation 2 by assuming $C$ to be equal to $(C_0/L^a)$ with $a$ a positive constant, $C_0$ for a system can be found out as

TABLE XX Common name, magnesium; class of material: (creep in) metal.

$\text{Ln}\, C_1 = 18.53$ ($C_1$ in $\text{s}^{-1}$); $\ln \sigma_c = 2.03$ ($\sigma_c$ in MPa).

Best fit: $n_{\text{pred}} = 26.904\,38 - 27\,121.4/T + 8337\,908/T^2$.

Linear fit: $n_{\text{pred}} = 4.759\,518 + 506.7561/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\text{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\text{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 820 | 0.555 | − 13.252 | − 14.593 | 10.12 | 0.65 |
| | 0.731 | − 12.285 | − 13.493 | 9.84 | 0.85 |
| | 0.974 | − 10.792 | − 11.978 | 10.99 | 2.69 |
| | 1.301 | − 8.802 | − 9.941 | 12.95 | 5.93 |
| 773 | 1.030 | − 12.739 | − 12.636 | 15.23 | 3.61 |
| | 1.265 | − 11.369 | − 11.278 | 0.81 | 3.18 |
| | 1.596 | − 9.425 | − 9.365 | 0.79 | 2.27 |
| | 2.079 | − 6.655 | − 6.578 | 0.63 | 0.87 |
| 723 | 1.135 | − 14.274 | − 13.403 | 1.15 | 5.36 |
| | 1.429 | − 12.734 | − 11.830 | 6.10 | 6.54 |
| | 1.727 | − 10.922 | − 10.240 | 7.10 | 5.92 |
| | 2.058 | − 8.997 | − 8.468 | 5.88 | 5.92 |
| 673 | 1.213 | − 15.547 | − 14.737 | 5.21 | 2.60 |
| | 1.592 | − 13.696 | − 12.836 | 6.98 | 4.70 |
| | 1.987 | − 11.752 | − 10.850 | 7.67 | 7.51 |
| | 2.274 | − 10.305 | − 9.415 | 8.64 | 9.84 |
| 623 | 1.527 | − 15.683 | − 15.423 | 1.65 | 0.64 |
| | 1.804 | − 14.218 | − 14.076 | 0.99 | 0.13 |
| | 2.129 | − 12.761 | − 12.499 | 2.05 | 2.63 |
| | 2.415 | − 11.184 | − 11.113 | 0.63 | 3.13 |
| 573 | 1.793 | − 16.719 | − 16.909 | 1.14 | 2.09 |
| | 2.100 | − 15.066 | − 15.385 | 2.12 | 1.79 |
| | 2.462 | − 13.215 | − 13.587 | 2.81 | 0.58 |
| | 3.032 | − 10.725 | − 10.756 | 0.29 | 6.05 |
| 523 | 2.220 | − 17.153 | − 17.956 | 4.69 | 4.46 |
| | 2.400 | − 16.157 | − 16.957 | 4.95 | 4.49 |
| | 2.636 | − 14.833 | − 15.656 | 5.55 | 4.73 |
| | 2.897 | − 13.490 | − 14.214 | 5.36 | 4.08 |

a unique constant that is independent of stress, temperature and grain size. By setting $C$ to be equal to $C_1$ or $C_2$, as the case may be, $C_0$ and $a$ can be calculated for the two cases of $\nu = (kT/h)$ and $10^{13}\ \text{s}^{-1}$, respectively. Owing to the paucity of data, numerical examples involving different grain sizes could not be provided.

## 6. Conclusions

From the results of the numerical verification carried out on the data pertaining to 27 systems, in all of which the isothermal isostructural $\ln \dot{\varepsilon}$–$\ln \sigma$ relationship was linear, the following conclusions/viewpoints have emerged.

1. There are reasons to believe that in the case of high-temperature creep and superplasticity (where diffusion and annealing effects cannot be ignored), normalization of the applied stress with respect to a reference stress is preferable to normalization with respect to an elastic constant, $E$. In addition, we prefer Equation 2 over Equation 1 because it helps reduce the number of empirical constants/mechanisms needed in the analysis. (This was the reason why Basinski [17] advocated normalization in the first place with respect to $E$ in situations where diffusion and annealing effects were unimportant.) For example, in the Al–Cu eutectic alloy (Table XXVIII) the results could be accounted for using a single activation energy, even though the original interpretation based on Equation 1 required two activation energies/rate-controlling processes [36]. There are physical reasons to believe that in this material the rate-controlling process would not have changed in the 100 °C temperature interval involved in the experiments [16, 18, 38].

2. The unique activation energy for each system reported here could either mean that there is a single rate-controlling flow mechanism, or that Equation 2 is a very good approximation for the weighted sum of two or more mechanisms, all of which make quantifiable contributions to the external flow rate. A preference between these two views can be exercised only through detailed microstructural and topological examination, which are not available in the references cited.

TABLE XXI Common name, tungsten; class of material: (creep in) metal.

$\ln C_1 = 14.23$ ($C_1$ in $s^{-1}$); $\ln \sigma_c = 1.94$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = 39.01418 - 160611/T + 1.9 \times 10^8/T^2$.

Linear fit: $n_{pred} = 4.657065 + 4254.674/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 3073 | 1.187 | − 15.248 | − 15.077 | 1.12 | 6.49 |
| | 1.310 | − 14.083 | − 14.203 | 0.85 | 4.02 |
| | 1.533 | − 12.479 | − 12.609 | 1.04 | 2.51 |
| | 1.689 | − 11.219 | − 11.493 | 2.44 | 0.01 |
| 2873 | 1.466 | − 14.297 | − 14.420 | 0.86 | 0.09 |
| | 1.632 | − 13.224 | − 13.351 | 0.96 | 0.29 |
| | 1.845 | − 12.377 | − 11.984 | 3.18 | 3.40 |
| | 1.996 | − 11.202 | − 11.010 | 1.71 | 1.57 |
| 2673 | 1.787 | − 14.279 | − 14.184 | 0.67 | 0.25 |
| | 1.934 | − 13.707 | − 13.321 | 2.82 | 2.80 |
| | 2.112 | − 12.647 | − 12.281 | 2.89 | 3.42 |
| | 2.249 | − 11.940 | − 11.476 | 3.89 | 4.89 |
| 2473 | 2.073 | − 15.221 | − 14.775 | 2.94 | 3.67 |
| | 2.203 | − 14.496 | − 14.054 | 3.05 | 4.58 |
| | 2.372 | − 13.353 | − 13.125 | 1.71 | 4.43 |
| | 2.525 | − 12.514 | − 12.277 | 1.89 | 5.83 |
| 2273 | 2.329 | − 15.221 | − 15.946 | 4.77 | 2.40 |
| | 2.555 | − 14.121 | − 14.681 | 0.96 | 0.08 |
| | 2.730 | − 13.347 | − 13.700 | 2.65 | 2.85 |
| | 2.849 | − 12.500 | − 13.035 | 4.28 | 2.48 |
| 2073 | 2.826 | − 15.326 | − 15.650 | 2.11 | 0.17 |
| | 3.054 | − 13.811 | − 14.212 | 2.90 | 0.27 |
| | 3.190 | − 12.729 | − 13.355 | 4.91 | 1.04 |
| 1873 | 3.192 | − 15.204 | − 14.872 | 2.18 | 7.59 |
| | 3.346 | − 14.019 | − 13.628 | 2.79 | 9.10 |
| | 3.455 | − 13.065 | − 12.741 | 2.48 | 11.28 |
| | 3.581 | − 12.218 | − 11.718 | 4.09 | 11.84 |

TABLE XXII Common name, comp-s10; class of material: metal matrix composite.

$\ln C_1 = 34.33$ ($C_1$ in $s^{-1}$); $\ln \sigma_c = 2.62$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = -94.9417 + 168517.6/T - 8.3 \times 10^7/T^2 + 1.4 \times 10^{10}/T^3$.

Linear fit: $n_{pred} = -8.36173 + 15152.84/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 823 | 0.585 | − 16.257 | − 16.924 | 4.11 | 9.25 |
| | 1.088 | − 10.688 | − 11.329 | 6.00 | 9.28 |
| 723 | 1.364 | − 16.389 | − 16.574 | 1.13 | 14.15 |
| | 1.712 | − 11.768 | − 11.505 | 2.24 | 17.62 |
| | 2.015 | − 6.837 | − 7.077 | 3.51 | 14.08 |
| 673 | 1.640 | − 16.240 | − 16.299 | 0.36 | 10.41 |
| | 2.016 | − 11.033 | − 10.295 | 6.69 | 16.43 |
| | 2.244 | − 6.400 | − 6.660 | 4.08 | 6.33 |
| 573 | 2.053 | − 17.546 | − 16.644 | 5.14 | 2.91 |
| | 2.349 | − 12.432 | − 11.493 | 7.56 | 6.07 |
| 473 | 2.467 | − 17.822 | − 18.078 | 1.44 | 7.09 |
| | 2.805 | − 12.093 | − 12.363 | 2.24 | 8.38 |
| 296 | 3.217 | − 17.827 | − 18.076 | 1.40 | 10.19 |
| | 3.358 | − 11.816 | − 11.665 | 1.28 | 15.09 |
| | 3.459 | − 6.761 | − 7.056 | 4.37 | 36.90 |

TABLE XXIII Common name, comp-s14; class of material: metal matrix composite.

Ln $C_1 = 37.06$ ($C_1$ in $s^{-1}$); ln $\sigma_c = 2.12$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = -8.11606 + 15443.41/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 823 | 0.734 | −17.682 | −18.884 | 6.80 |
| | 1.098 | −14.386 | −15.005 | 4.30 |
| 773 | 1.232 | −17.314 | −17.303 | 0.06 |
| | 1.516 | −14.086 | −13.935 | 1.07 |
| 723 | 1.645 | −17.358 | −16.085 | 7.33 |
| | 1.971 | −12.492 | −11.763 | 5.84 |
| | 2.191 | −7.712 | −8.850 | 14.75 |
| 573 | 2.299 | −17.727 | −18.624 | 5.06 |
| | 2.552 | −13.575 | −13.870 | 2.17 |
| 296 | 3.473 | −17.304 | −17.473 | 0.97 |
| | 3.582 | −12.939 | −12.672 | 2.07 |
| | 3.666 | −8.702 | −8.966 | 3.03 |

TABLE XXIV Common name, comp-s7; class of material: metal matrix composite.

Ln $C_1 = 22.67$ ($C_1$ in $s^{-1}$); ln $\sigma_c = 2.96$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = 20.70567 - 20462.9/T + 9492199/T^2$.

Linear fit: $n_{pred} = -22.9530 + 23437.04/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 823 | 0.068 | −18.570 | −20.757 | 11.78 | 55.59 |
| | 0.722 | −12.567 | −14.313 | 13.89 | 63.11 |
| 773 | 0.724 | −17.119 | −15.843 | 7.45 | 43.34 |
| | 1.118 | −12.983 | −11.856 | 8.69 | 47.65 |
| 573 | 1.766 | −16.271 | −15.362 | 5.58 | 23.99 |
| | 2.129 | −10.813 | −10.308 | 4.67 | 26.24 |
| 473 | 2.198 | −17.785 | −18.426 | 3.60 | 32.27 |
| | 2.620 | −10.517 | −10.038 | 4.56 | 16.93 |
| | 2.817 | −5.828 | −6.114 | 4.91 | 20.90 |
| 296 | 2.979 | −17.391 | −17.505 | 0.66 | 1.16 |
| | 3.069 | −12.375 | −12.188 | 1.51 | 1.84 |
| | 3.164 | −6.335 | −6.457 | 1.94 | 14.05 |

TABLE XXV Common name, comp-s4; class of material: metal matrix composite.

Ln $C_1 = 36.87$ ($C_1$ in $s^{-1}$); ln $\sigma_c = 2.40$ ($\sigma_c$ in MPa).

Best fit: $n_{pred} = 31.55327 - 25729.5/T + 9070704/T^2$.

Linear fit: $n_{pred} = -12.4240 + 17208.55/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 773 | 0.539 | −16.952 | −19.718 | 16.31 | 23.30 |
| | 0.869 | −13.597 | −15.272 | 12.31 | 28.29 |
| | 0.991 | −11.421 | −13.629 | 19.43 | 25.09 |
| 673 | 1.356 | −15.352 | −13.319 | 13.24 | 14.62 |
| | 1.470 | −13.554 | −11.793 | 12.99 | 14.38 |
| 473 | 2.292 | −16.500 | −16.615 | 0.70 | 4.73 |
| | 2.497 | −13.043 | −12.987 | 0.43 | 5.16 |
| 296 | 2.988 | −17.154 | −17.213 | 0.34 | 8.74 |
| | 3.082 | −12.780 | −12.669 | 0.88 | 12.2 |
| | 3.130 | −10.235 | −10.354 | 1.16 | 18.63 |

TABLE XXVI Common name, WC–Co; class of material: metal matrix composite.
Ln $C_1$ = 34.04 ($C_1$ in $s^{-1}$); ln $\sigma_c$ = 6.68 ($\sigma_c$ in MPa).
Best fit: $n_{pred} = 5.682\,398 - 4837.62/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1373 | 4.311 | − 10.756 | − 10.834 | 0.72 |
| | 4.641 | − 9.912 | − 10.120 | 2.10 |
| | 5.092 | − 9.054 | − 9.146 | 1.03 |
| | 5.456 | − 8.391 | − 8.360 | 0.37 |
| | 5.707 | − 7.722 | − 7.821 | 1.29 |
| 1423 | 3.899 | − 10.716 | − 10.671 | 0.42 |
| | 4.217 | − 9.945 | − 9.943 | 0.02 |
| | 4.531 | − 9.158 | − 9.228 | 0.77 |
| | 4.792 | − 8.500 | − 8.633 | 1.56 |
| | 5.133 | − 7.747 | − 7.853 | 1.36 |
| | 5.527 | − 6.966 | − 6.955 | 0.17 |
| 1473 | 3.720 | − 9.927 | − 10.120 | 1.95 |
| | 3.930 | − 9.059 | − 9.615 | 6.14 |
| | 4.093 | − 8.530 | − 9.224 | 8.15 |
| | 4.388 | − 7.849 | − 8.516 | 8.49 |
| | 4.840 | − 6.940 | − 7.433 | 7.12 |
| | 5.209 | − 6.413 | − 6.548 | 2.10 |
| | 5.969 | − 4.500 | − 4.726 | 5.00 |

TABLE XXVII Common name, $Ni_3Si$; class of material: intermetallic.
Ln $C_1$ = 45.35 ($C_1$ in $s^{-1}$); ln $\sigma_c$ = 3.52 ($\sigma_c$ in MPa).
Best fit: $n_{pred} = -17.7576 + 26\,636.38/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $s^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $s^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 1323 | 2.762 | − 6.955 | − 7.011 | 0.81 |
| | 3.413 | − 5.598 | − 5.465 | 2.39 |
| | 3.625 | − 4.852 | − 4.961 | 2.23 |
| 1353 | 2.426 | − 6.206 | − 6.201 | 0.08 |
| | 3.103 | − 5.017 | − 4.895 | 2.45 |
| | 3.839 | − 3.403 | − 3.475 | 2.14 |
| 1373 | 2.126 | − 5.566 | − 5.659 | 1.68 |
| | 2.649 | − 4.845 | − 4.800 | 0.94 |
| | 3.157 | − 3.907 | − 3.960 | 1.50 |

3. For the systems examined, Equation 2 is a very good description of the flow kinetics. For the isostructural, isothermal case, $n$ is a constant and for each system $n$ as a function of $(1/T)$ has also been obtained. $C_1$ (or $C_2$), $\sigma_c$ and $Q$ are unique constants for a system in a given microstructural condition. The strain rate–stress relationship at any temperature within the given interval can be determined from the above material constants, the relation between $n$ and $(1/T)$ and the method of interpolation. Depending on whether the $n$ versus $(1/T)$ relationship is linear, quadratic or cubic, the number of empirical constants needed in the analysis is 5, 6 or 7 per system of a given microstructure. Except for three systems, in which the temperature range covered was rather wide, in all other cases a linear relationship between $n$ and $(1/T)$ was sufficient for an accurate estimation of the strain rate. Evidently, the number of empirical constants needed in this analysis is less than 10, the number of constants needed if the experimental data pertaining to any two phase material were to be interpreted in terms of Equation 1.

A generic analysis of the kind presented here can be of use in design as it is based on an "archetypal rate equation" that is independent of the details of the physical processes of deformation involved.

4. The next step should be to give physical meaning to the empirical constants of Equation 2 treating it either as a rigorous representation of steady-state high-temperature deformation or as an approxima-

TABLE XXVIII Common name, Al–Cu; class of material: superplastic metallic alloy.

$\ln C_1 = 14.26$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 2.87$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -1.75418 + 2674.094/T$

| Temperature (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) |
|---|---|---|---|---|
| 813 | − 0.647 | − 10.376 | − 10.658 | 2.71 |
| | − 0.181 | − 9.704 | − 9.941 | 2.44 |
| | 0.258 | − 9.039 | − 9.267 | 2.52 |
| | 0.821 | − 8.149 | − 8.401 | 3.10 |
| 803 | − 0.251 | − 10.388 | − 10.421 | 0.30 |
| | 0.214 | − 9.701 | − 9.686 | 0.16 |
| | 0.625 | − 9.025 | − 9.037 | 0.13 |
| | 1.168 | − 8.168 | − 8.182 | 0.16 |
| 793 | 0.025 | − 10.453 | − 10.353 | 0.96 |
| | 0.447 | − 9.766 | − 9.670 | 0.99 |
| | 0.831 | − 9.091 | − 9.048 | 0.48 |
| | 1.390 | − 8.217 | − 8.144 | 0.89 |
| 783 | 0.157 | − 10.455 | − 10.512 | 0.54 |
| | 0.634 | − 9.789 | − 9.718 | 0.73 |
| | 1.073 | − 9.113 | − 8.989 | 1.36 |
| | 1.550 | − 8.224 | − 8.198 | 0.32 |
| 773 | 0.482 | − 10.481 | − 10.339 | 1.35 |
| | 0.872 | − 9.838 | − 9.673 | 1.68 |
| | 1.313 | − 9.059 | − 8.921 | 1.52 |
| | 1.753 | − 8.247 | − 8.171 | 0.93 |
| 753 | 0.851 | − 10.516 | − 10.440 | 0.72 |
| | 1.286 | − 9.827 | − 9.659 | 2.16 |
| | 1.646 | − 9.149 | − 9.011 | 1.51 |
| | 2.105 | − 8.239 | − 8.187 | 0.64 |
| 713 | 1.532 | − 10.511 | − 10.665 | 1.46 |
| | 1.910 | − 9.815 | − 9.910 | 0.97 |
| | 2.223 | − 9.141 | − 9.286 | 1.58 |
| | 2.703 | − 8.220 | − 8.327 | 1.30 |

TABLE XXIX Common name, Pb–Sn; class of material: superplastic metallic alloy.

$\ln C_1 = 7.69$ ($C_1$ in $\mathrm{s}^{-1}$); $\ln \sigma_c = 4.10$ ($\sigma_c$ in MPa).

Best fit: $n_{\mathrm{pred}} = -9.53195 + 7881.599/T - 1300043/T^2$.

Linear fit: $n_{\mathrm{pred}} = -0.32142 + 917.4339/T$

| $T$ (K) | Ln $\sigma$ (observed) ($\sigma$ in MPa) | Ln $\dot{\varepsilon}$ (observed) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Ln $\dot{\varepsilon}$ (predicted) ($\dot{\varepsilon}$ in $\mathrm{s}^{-1}$) | Error in prediction (%) | Error in prediction when $n$ versus $1/T$ is assumed to be linear (%) |
|---|---|---|---|---|---|
| 443 | − 0.941 | − 8.942 | − 9.169 | 2.53 | 8.99 |
| | − 0.271 | − 8.025 | − 8.024 | 0.61 | 6.85 |
| | 0.033 | − 7.345 | − 7.576 | 3.15 | 9.49 |
| | 0.584 | − 6.672 | − 6.675 | 0.04 | 6.07 |
| | 0.940 | − 5.751 | − 6.092 | 5.93 | 12.22 |
| 413 | 0.378 | − 8.990 | − 8.734 | 2.84 | 4.09 |
| | 0.848 | − 8.080 | − 7.829 | 3.12 | 4.32 |
| | 1.508 | − 6.715 | − 6.554 | 2.40 | 3.56 |
| | 2.091 | − 5.724 | − 5.430 | 5.15 | 6.21 |
| | 2.231 | − 5.114 | − 5.159 | 0.88 | 0.21 |
| 383 | 0.319 | − 10.352 | − 10.534 | 1.75 | 2.27 |
| | 0.618 | − 9.713 | − 9.880 | 1.72 | 2.23 |
| | 0.959 | − 9.028 | − 8.836 | 1.20 | 2.63 |
| | 1.361 | − 8.050 | − 8.257 | 2.56 | 1.19 |
| | 1.779 | − 7.418 | − 7.345 | 0.99 | 4.43 |
| 353 | 1.342 | − 9.748 | − 9.637 | 1.13 | 3.54 |
| | 1.601 | − 8.990 | − 9.027 | 0.41 | 1.95 |
| | 1.957 | − 8.161 | − 8.185 | 0.30 | 1.93 |
| | 2.308 | − 7.414 | − 7.356 | 0.79 | 2.84 |
| | 2.648 | − 6.688 | − 6.552 | 2.04 | 3.89 |
| 323 | 1.459 | − 10.377 | − 10.779 | 3.87 | 1.06 |
| | 1.748 | − 9.712 | − 10.051 | 3.49 | 0.81 |
| | 2.135 | − 8.971 | − 9.077 | 1.18 | 1.25 |
| | 2.450 | − 8.054 | − 8.282 | 2.83 | 0.57 |
| | 2.842 | − 7.385 | − 7.295 | 1.22 | 3.10 |

tion of a more complicated equation. Some speculations with regard to the physical meaning of $\sigma_c$ have already emerged [2, 4, 39].

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